XIX. *On the parallax of the fixed stars.* *By* J. F. W. HERSCHEL, *Esq. M. A. Sec. R. S. Communicated January* 19, 1826.

Read March 9 and 16, 1826.

THE determination of the existence and amount of annual parallax in the fixed stars, was the object which originally drew the attention of astronomers to the examination and measurement of double stars, upon a principle suggested originally by GALILEO, and improved on and more fully developed by my Father in one of his earliest communications to the Royal Society, Phil. Trans. 1782; according to which, two stars placed very nearly in a line with each other and with our system, ought, if situated at very different distances, in the line of sight, to be subject to periodical variations in their apparent distance from each other, according as the earth in its annual motion approaches to or recedes from that point in the plane of the ecliptic, where the line joining them, prolonged, would meet it. The difficulty of determining the distance of two stars of a double star with the necessary degree of exactness for this delicate purpose, has however hitherto put a stop to this enquiry; and, as it is now rendered extremely probable that the parallax at least of the generality of stars is much below 1″, must continue to do so, until some very great improvement of micrometers shall enable us to measure tenths of seconds with as great certainty as we at present can units.

I do not find that it has been noticed, however, that parallax

must occasion a periodical change in the angle of position, as well as in the distance of the two stars composing a double star, and that this variation is much more susceptible of ready and exact appreciation with our present micrometers than that of their distance. To render this sensible, we need only remark, that the effect of parallax is to cause each star to describe (apparently) a small ellipse in the heavens, the major axis of which is parallel to the ecliptic, and the minor in the direction of a secondary to that great circle, the true place of the stars being in the centre. Were these ellipses of the same magnitude, for each of two contiguous stars, the line joining their apparent places (which are necessarily homologous points in the circumferences of each) would preserve its parallelism at all times; but as the axes of the ellipses are reciprocally as the distances of the stars, that parallelism cannot obtain when the stars are situated at very unequal distances from the earth, and an alternate increase and decrease of the angle of position made by this line with any fixed direction must be the necessary consequence.

To estimate the extent of this variation, let us conceive two stars so situated as to have their apparent line of junction in the direction of a secondary to the ecliptic, and therefore at right angles to the major axes of their parallactic ellipses— let their distances from us be such that the nearer one shall have a parallax of 1″, and the farther no appreciable amount of it. Also, let their apparent angular distance from each other be 5″. It is evident that the variation alluded to will equal the angle subtended by a line of 1″ in length, at a point 5″ distant from its middle, that is, to 11° 25′.

Now this is a quantity which is quite beyond all conceivable limits of error of observation in the measurement of double

stars, and for stars nearer than 5″ the amount is of course proportionally greater. Thus for two stars, at only 1″ distance from each other, of which the one is affected by parallax to the amount of 1″, and the other not all, the annual variation in position will amount to upwards of 53°.

When the distance between two stars amounts to no more than 3″, and in many cases even when they are still nearer, Mr. SOUTH's and my own experience in the use of the position micrometer, as well as Mr. STRUVE's numerous and excellent measures recorded in the Dorpat Observations, lead me to believe that a single degree in the angle of position is a quantity distinctly appreciable in the mean of several sets of measures carefully taken on different nights of observation, even when a considerable inequality in the stars, or other unfavourable circumstances exist; and were observations continued for a series of years at the proper times, and made with the care so delicate a subject of research would peculiarly call for, there can be no doubt that a greater degree of precision might be obtained; and it certainly seems not too much to assume, that half that quantity, or 30′ of variation in the angle of position, *if regularly periodical*, must ultimately be detected. This conclusion appears warranted by the interesting re-examination by Mr. SOUTH of stars pointed out in our joint paper on Double Stars (Ph. Tr. 1824. iii.) as having probably a relative angular motion, or as otherwise remarkable, lately communicated by him to the Royal Society, (See Part i. of the present vol.) in which we find instances of coincidence between calculated and computed motions falling within the limit assumed at the outset—too frequent, and too remarkable to be the effect of mere accident.

Now, the tangent of 30′ to a radius of 3″ corresponds to a

subtense of $0''.026$, or $\frac{1}{40}$th of a second, so that a difference of parallaxes to the amount of a 40th of a second, existing in the two stars of a double star so circumstanced, could scarcely escape detection; and that even much less quantities than this, under favourable circumstances, might be rendered sensible, I think may fairly be concluded, when we consider that in this estimate, the data are certainly assumed within bounds. No account is here taken of the improvements in the position micrometer, which may reasonably be expected, because the object at present is only to appreciate the degree of delicacy which the method now proposed may lay claim to, with our present instruments and habits of observing, and to compare it with those which have hitherto been resorted to in the investigation of parallax.

In selecting stars for examination, it appears to me that we ought by no means to confine ourselves, by assuming it as a universal law that the brightest stars are the nearest to us. From what we know of the variety of nature and the enormous differences in point of magnitude between the bodies of our own system, it seems improbable that the real magnitude and brightness of the stars should be confined within narrow limits. Their distances are equally undetermined; nor have we any reason whatever to conceive these two elements related to each other. There is not therefore the slightest *a priori* improbability, in supposing that among stars of apparently equal lustre, the greatest diversity of distance may exist, or that innumerable of the minutest stars visible in telescopes may be nearer to us than any of those of the first magnitude; and consequently, that that delicate element in search of which astronomers have exhausted

refinement, may with nearly or quite equal probability of success be sought among stars of far inferior magnitudes. The proper motions of the stars afford an argument from analogy—they bear no relation to their apparent lustre, and by far the greatest proper motions known belong to stars low in the scale of magnitudes.

If these remarks have any foundation, it must be obvious that we ought not to be deterred from the research of parallax by the smallness of the stars composing a double star, or their approach to equality. However, we should except here stars in, or very near the milky way, below the 7th or 8th magnitudes, because the probable *laminar* form of this great sidereal stratum affords a presumption almost amounting to certainty, that minuteness is *here, on the average*, an effect of distance. But, on the other hand, such large stars as β Orionis, which are situated in, or near the borders of the milky way, with small ones near enough to constitute them double stars, have an additional claim to examination, from the additional probability thus afforded of being favourably placed for the detection of parallax: and moreover, this consideration affords plausible grounds for a belief, that, in situations remote from the milky way, minuteness, *on the average*, is *not* the effect of distance.

It is hardly necessary to insist on the great advantages presented by the method here proposed, in its complete exemption from those instrumental errors depending on unsteadiness, erroneous graduation, and expansion, and from all that uncertainty on the score of refraction, and any doubts still remaining as to the magnitudes of the constants of aberration, nutation, &c., which so much embarrass astronomers.

A good telescope and a good micrometer are all the instruments required. This advantage is really incalculable. Instead of confining our attention to one or two principal stars, it puts an almost unlimited range of objects in our power, by enabling us to employ in this research the largest telescopes, and thus easily obtain measures of those stars, which, from their faintness, must present insuperable difficulties with instruments of ordinary apertures.

In selecting objects for examination by this method, we must be chiefly guided by their angles of position and distances. Taking such whose distances are below $15''$, (which, on the supposition of $\frac{1}{2}^{\circ}$ periodical variation of position, corresponds to $\frac{1}{8}$ of a second of relative parallax in stars properly situated), the angle of position ought to be such, that the line joining the two stars shall point as nearly as may be, to the pole of the ecliptic. Ten, twenty, or even thirty degrees of deviation either way from this direction, will however not materially vitiate the application of this method to stars near the ecliptic, while, for such as have considerable latitudes, proportionally greater deviations may be allowed, and within thirty degrees of the pole of the ecliptic this element is of comparatively small moment.

In general, to ascertain whether any double star is or is not favourably situated for the application of this method, we must (if we would take up the problem on strict mathematical grounds), proceed as follows:

Let l represent the longitude, and $+\lambda$ the N. latitude of the star, σ its *angle of situation*,* or the angle included at the

* This angle in many astronomical books is called the angle of position, but as in speaking of double stars we have always hitherto called the angle made by the

star by great circles joining it and the north poles of the equinoctial and ecliptic respectively, σ being considered positive for stars in the western hemisphere (or that whose pole is the point ♈), and negative for stars in the eastern, whose pole is ♎. Let also ☉ represent the sun's longitude at any time, and call a the maximum semi-annual parallax or the angle (expressed in seconds), which the radius of the earth's orbit would subtend if perpendicularly presented to an eye at the distance of the star. It is obvious then that a will represent the major semi-axis of the star's parallactic ellipse, and $a \,.\, \sin. \lambda$ the minor, so that its excentricity $= \sqrt{a^2 - a^2 . \sin. \lambda^2}$ $= a \,.\, \cos. \lambda$, which if we call $a\,e$, we have $e = \cos. \lambda$. The star will appear to describe this ellipse in the direction *npsf*. Its motion in it will however not be uniform, but equal areas will be described in equal times about its centre (or the star's mean place): this is evident, because the area described by the star in the parallactic ellipse round its centre is the orthographic projection on the surface of the heavens of that described by the earth round the sun in its orbit. This consideration gives us at once, the equation

$$\tan. \theta = - \sin. l \,.\, \text{cotan.} (\odot - l); \qquad (1)$$

where θ represents the elongation of the star in its ellipse from the eastern extremity of its major axis reckoned in the direction *npsf*.

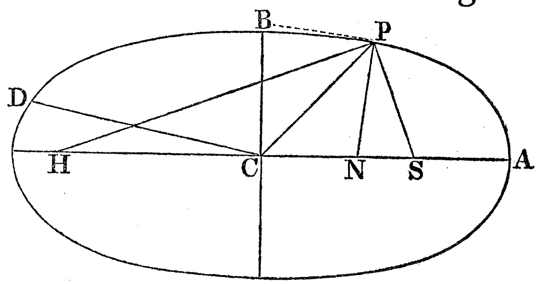
Let π be the angle of position of the small star, π being reckoned from a parallel to the equinoctial, in the same direction *npsf* and from the east, so that the *nf* quadrant shall

line joining the two stars with the parallel, by this name, it becomes necessary to distinguish them, and the expression used in the text is perhaps also more correct than that in common use.

correspond to values of π between 0 and 90°, the np to values between 90° and 180° and so on. It is evident, then, that when the large star is in the point of its apparent orbit immediately in the line joining the mean places of both, its elongation, or the value of θ corresponding to this situation will be $\pi - \sigma$.

Since it is only the difference of parallaxes which this method can render sensible, we may suppose the small star a fixed point, and since the dimensions of the parallactic ellipse may be supposed small in comparison with the distance of the small star, two tangents drawn from the latter to the circumference of the former, and which, of course, mark the situation of the large one in its apparent orbit where parallax has the greatest effects in opposite senses on the angle of position, will nearly meet it in the two extremities of a diameter conjugate to that passing through the small star.

To determine the direction of the conjugate diameter, we must have recourse to the general equations of the ellipse from its centre with polar co-ordinates. Thus r being the length of any semi-diameter CP, whose angle of elongation from CA, is ACP $= \varphi$ and r', that of its semiconjugate CD, the angle ACD being



called φ', we have, H and S being the foci, and calling CN the abscissa from the centre, x, by a property of the conic sections, $SP = a - ex = a - er \cos. \varphi$; $HP = a + ex = a + er. \cos. \varphi$; whence, $\quad SP . HP = CD^2 = r'^2 = a^2 - e^2 r^2 . \cos. \varphi^2$.

But we have also, $\quad r^2 = \frac{a^2 (1 - e^2)}{1 - e^2 . \cos. \varphi^2}$; $r'^2 = \frac{a^2 (1 - e^2)}{1 - e^2 . \cos. \varphi'^2}$.

Eliminating from these three equations, r and r', we get the relation required between φ and φ', viz.

$$\cos.\ \varphi'^2 = \frac{\sin.\ \varphi^2}{1-(2-e^2)\ e^2.\ \cos.\ \varphi^2}$$

This equation will however be reduced to a much more convenient form for our present purpose by a transformation, viz.

$$\tan.\ \varphi'^2 = \frac{\sin.\ \varphi'^2}{\cos.\ \varphi'^2} = \frac{1-\cos.\ \varphi'^2}{\cos.\ \varphi'^2}$$
$$= \frac{(1-e^2)^2.\ \cos.\ \varphi^2}{\sin.\ \varphi^2}$$

that is simply

$$-\tan.\ \varphi' = (1-e^2).\ \cot.\ \varphi$$

the sign — being prefixed in extracting the root, became in the ellipse C P and C D lie in different quadrants. Now in the case before us we have

$$\varphi = \pi - \sigma \text{ and } \varphi' = \theta\text{; so that}$$
$$\tan.\ \theta = -(1-e^2).\ \text{cotan.}\ (\pi-\sigma)$$

But $(1-e^2) = \sin.\ \lambda^2$, and by equation (1) $-\tan.\ \theta = \sin.\ l.$ $\text{cotan.}\ (\odot - l)$. Hence substituting, we get

$$\sin.\ \lambda^2.\ \tan.\ (\odot - l) = \sin.\ l.\ \tan.\ (\pi-\sigma); \qquad (2)$$

This equation gives at once the value of $\odot$ the sun's longitude, and therefore (by consulting an ephemeris) the time of year sought, by an exceedingly simple process adapted to logarithmic computation.

The actual advantage or disadvantage in point of situation in the case of any particular star, is expressed by the magnitude of the whole change in the angle of position produced by parallax, *i. e.* by the angle subtended by the ellipse at the small star, or, (calling the distance of the latter from its centre δ) on the original supposition of the whole effect being small, by the expression $P = \frac{2r'}{\delta}.\ \sin.\ (\varphi' - \varphi)$. Now if we

substitute for r' and sin. $(\phi' - \phi)$ their values in terms of ϕ, or $\pi - \sigma$ deduced from the equations already stated, by proper eliminations, we find

$$P = \frac{2a}{\delta} \cdot \sqrt{1 - \cos. \lambda^2. \cos. (\pi - \sigma)^2} \qquad (3)$$

whence the maximum effect of parallax on the angle of position of any double star is readily computed; the expression is inconvenient for logarithmic computation, but this inconvenience is obviated by the very obvious substitution

$$\cos. \lambda. \cos. (\pi - \sigma) = \cos. M; \; P = \frac{2a}{\delta}. \sin. M. \qquad (4$$

We also have

$$2a = \delta. \frac{P}{\sin. M} = \delta. \frac{\sin. P}{\sin. M} \qquad (5)$$

from which we may deduce the value of $2a$ the maximum of parallax when the total effect (P) on the angle of position is known.

If we take $P = 30'$ we have $2a = \delta. \frac{\sin. 30'}{\sin. M}$ (6)

which expresses the amount of annual parallax in any proposed star which will be indicated by a periodical variation of $30'$ in the angle of position. The smallness of this amount is a criterion (mathematically speaking) of the favourable or unfavourable nature of the individual star for researches of this kind.

By these theorems the proper times of year for observation, and the amount of difference of parallax, appreciable by a variation of $30'$ in the angle of position, have been computed for such stars among those observed by Mr. SOUTH and myself, in the paper above alluded to, as appear favourable to the application of the common wire micrometer to this enquiry, in the method now proposed. As the number of known double stars increases, others may easily be added to the list. The Catalogue of 460 double Stars observed by

Mr. South, and lately communicated to the Royal Society, will furnish a great many; and Mr. Struve's immense collection, amassed in his reviews with the large refractor, as well as a collection of minute double stars encountered in my own 20-feet sweeps, and which will shortly be published, doubtless many more; so that it is rather intended to give the following list as a specimen of a more complete one, than as including all, or nearly all, the stars it is desirable to examine with this view. Meanwhile, to enable others not conversant with algebraic symbols to extend the list for themselves, if so inclined, I shall here set down the whole work of calculation for one star.

Calculation for 35 Piscium. R. A. $0^h\ 6^m = \alpha = 1°\ 30'$, Decl. $7°\ 49'$ N. Long. $= l = 4°\ 24'$. Lat. $= \lambda = +\ 6°\ 32'$; Angle of position $= \pi = 298°\ 4'$; Distance $= \delta = 12''.5$.

sin. obliq^y.	+ 9·60012	(2) tan. $(\pi - \sigma)$	−11·09285	(1) cos. lat.	+ 9·99717
cos. R. A.	+ 9·99985	sin. long.	+ 8·88490	(2) cos. $(\pi - \sigma)$	+ 8·90574
		(add)	−19·97775	(3) (add) cos. M	+ 8·90291
(1) ar. comp. cos. lat.	+ 0·00283	(1) 2 × log. sin. lat.	+ 8·11214	log. δ	+ 1·04798
				tan. 30′	+ 7·94086
sin. σ	+ 9·60280	(subtr.) tan. $(\odot - l)$	−11·86561		
σ	23° 27′	$\odot - l$	270° 47′	(add)	+ 8·98884
π	298 4	l	4 24	(3) sin. M	+ 9·99861
$\pi - \sigma$	274 37	$\odot$	275 11	(subtr.) log. 2 a	+ 8·90023
		$\odot + 6^s$	95 11	2 a	0·098
		Dates by Nautical Almanack .	Dec. 27 June 27		

If we wish to avoid the computation of the latitudes and longitudes which this supposes known, they may be taken with sufficient precision from any good map, or from a large celestial globe, as exactness is not required for this purpose in these elements.

Specimen of a List of Stars

favourably situated for the Investigation of Parallax by the Method proposed in this Paper.

General Number in Messrs. H. and S's. Observations.	Stars' Name, &c.	R. A. for 1820.	Declination for 1820.	Times of the year most proper for Observation.	Amount of annual Parallax indicated by a periodical variation of 30′ in the angle of pos.
		h. m.	° ′		
1	35 Piscium	0 6	7 49 N	June 27, December 27..	0″.098
20	γ Arietis	1 44	18 25 N	January 31, August 3 ..	.085
25	α Piscium	1 53	1 53 N	January 18, July 19....	.047
38	32 Eridani.....	3 45	3 30 S	February 12, August 15	.071
39	ε Persei	3 46	39 29 N	February 19, August 23	.081
46	55 Eridani.....	4 35	9 9 S	March 6, September 8..	.103
47	ω Aurigæ......	4 47	37 36 N	March 5, September 7..	.069
53	Rigel	5 6	8 25 S	February 25, August 29	.084
55	118 Tauri.....	5 18	25 0 N	March 10, September 13	.052
56	32 Orionis.....	5 21	5 48 N	March 7, September 9..	.013
59	33 Orionis.....	5 22	3 9 N	March 6, September 8..	.020
67	ζ Orionis	5 32	2 3 S	March 18, September 20	.025
366	41 Aurigæ.....	5 58	48 44 N	March 21, September 23	.077
69	8 Monocerotis ..	6 14	4 41 N	March 21, September 24	.136
76	38 Geminorum .	6 44	13 24 N	April 1, October 4.....	.049
80	δ Geminorum ..	7 9	22 18 N	April 5, October 9.....	.064
88	11 Cancri	7 58	28 0 N	April 16, October 19...	.041
93	φ^2 Cancri......	8 16	27 31 N	April 19, October 23...	.051
94	18 Bode, Hydræ	8 26	7 15 N	April 26, October 29 ..	.096
96	144 of 145.....	8 39	71 27 N	April 20, October 23...	.076
98	57 Cancri	8 43	31 16 N	April 27, October 30...	.020
99	17 Hydræ	8 47	7 17 S	May 12, November 14 .	.053
102	Cancri 194	8 57	23 42 N	April 30, November 2..	.067
114	Leonis 145	10 11	7 22 N	May 23, November 24..	.060
128	90 Leonis	11 25	17 48 N	June 4, December 6....	.039
133	65 Ursæ.......	11 46	47 29 N	May 21, November 23..	.035
134	2 Comæ.......	11 55	22 28 N	May 17, November 18..	.038
147	118 of 145.....	12 25	75 46 N	June 4, December 6....	.053
152	Str. 422	12 40	4 48 N	June 30, December 30..	.089
155	Str. 424.....	12 44	16 0 N	May 29, November 30..	.072
161	54 Virginis	13 4	17 51 S	January 13, July 15....	.060
167	81 Virginis	13 28	6 57 S	January 14, July 15....	.038
73	98 of 145......	14 5	6 14 N	January 15, July 17....	.054

Specimen of a List of Stars—*continued.*

General Number in Messrs. H. and S's. Observations.	Star's Name, &c.	R. A. for 1820.	Declination for 1820.	Times of the year most proper for Observation.	Amount of Annual Parallax indicated by a periodical variation of 30′ in the angle of pos.
		h. m.	° ′		
176	Str. 456......	14 13	6 56 S	January 23, July 26....	0″.061
177	Str. 457......	14 14	9 16 N	January 14, July 16....	.064
188	39 Bootis......	14 44	49 27 N	April 2, October 5.....	.040
193	44 Bootis......	14 58	48 21 N	February 10, August 13	.020
194	Str. 474......	14 59	9 55 N	February 2, August 4..	.042
201	η Coronæ B....	15 16	30 57 N	January 28, July 31....	.014
205	δ Serpentis?....	15 26	11 9 N	February 6, August 9..	.027
206	Libræ 178... .	15 30	8 11 S	February 11, August 14	.104
211	II. 85.........	15 47	1 39 S	February 7, August 10.	.085
212	III. 103.......	15 48	3 56 N	January 31, August 2..	.132
228	g 5 Ophiuchi..	16 15	23 1 S	February 25, August 29	.036
240	——	16 46	19 15 S	March 3, September 5..	.064
245	39 Ophiuchi...	17 7	24 5 S	March 8, September 10.	.111
262	100 Herculis...	18 1	26 5 N	March 22, September 24	.125
265	I. 86..........	18 12	25 28 N	March 21, September 24	.040
269	39 Draconis...	18 21	58 42 N	April 28, November 1..	.031
271	——	18 30	41 7 N	March 22, September 25	.053
274	——	18 36	10 39 S	March 28, October 1...	.049
280	——	18 42	10 47 N	April 5, October 8.....	.042
287	——	18 58	6 53 N	April 2, October 5.....	.077
295	III. 57........	19 19	20 46 N	April 7, October 10....	.062
306	π Aquilæ......	19 41	11 22 N	April 8, October 11....	.021
311	ε Draconis.....	19 49	69 48 N	April 20, October 23...	.023
312	ψ Cygni.......	19 51	51 38 N	January 8, July 9......	.038
313	I. 96..........	19 56	35 32 N	May 19, November 20..	.022
317	II. 96.........	20 3	0 19 N	April 30, November 2..	.045
320	I. 95..........	20 14	54 48 N	January 13, July 15....	.035
323	ϱ Capricorni II..	20 20	18 24 S	April 22, October 26..	.036
326	——	20 32	38 5 N	May 26, November 27.	.085
343	Str. 751......	22 16	65 50 N	March 2, September 4..	.033
349	Aquarii 213....	22 34	9 11 S	May 28, November 28.	.031
352	——	22 59	31 51 N	February 24, August 28	.076
354	94 Aquarii....	23 10	14 26 S	May 20, November 21..	.133
356	107 Aquarii...	23 37	19 41 N	June 3, December 5....	.045
359	σ Cassiop......	23 50	54 45 N	January 11, July 13....	.026

If I am not mistaken, then, we have here a method of investigating parallax very much more delicate than any which has yet been proposed—open to no insuperable objections—demanding no laborious reductions—and depending on a branch of practical astronomy, which is now happily cultivated with the assiduity it so richly deserves. It may not then be too much to hope, that in a few years we shall have data for a positive decision, respecting at least a certain number of individual stars, and thus be enabled to form a much better judgement as to the probable maximum of parallax in any. Meanwhile, the number of stars to which the method is applicable must not be judged of from the specimen above given. The only position micrometer of which I have any knowledge from practice is the wire micrometer; but the double image micrometer, if used in a manner, for the knowledge of which I am indebted to Captain KATER, and which consists in bringing the 1st image of one star almost, but not quite in contact with the 2nd image of the other, and in the same straight line, presents the singular advantage of an attainable exactness proportional to the distance of the stars; and by employing angles of position thus measured, stars of the 4th, 5th and 6th classes are rendered equally, or more available than those of the 1st, 2nd and 3rd; so that our range of objects becomes, by the use of this instrument in this manner really unlimited.

It is possible, that by some an apology may be thought necessary for a communication like the present—offered as it is to the Royal Society as a mere proposal—not followed out into actual practice. A variety of other pursuits, however, and circumstances less under my own controul, have

hitherto prevented, and will in all probability continue to prevent, my engaging actively in the very extensive and laborious series of observations required for this purpose. Should I do so in future, to which however I by no means intend to pledge myself, my progress must of necessity be extremely slow. But should the method itself appear really to possess the advantages I am inclined to ascribe to it, practical astronomers I am sure will hardly impute its publication, even unaccompanied with observations, as a fault; and if their energy and perseverance should anticipate me in its application, I hope I should be the first to acknowledge that the merit of the discovery of parallax must rest with him, who, whatever be the method he may pursue, shall first point out the star in which it exists, and establish it to the satisfaction of astronomers by unequivocal observations.

J. F. W. H.

London, Dec. 8, 1825.